Fault detection for linear parameter varying systems under changes in the process noise covariance

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Abstract: Detecting changes in the eigenstructure of linear systems is a comprehensively investigated subject. In particular, change detection methods based on hypothesis testing using Gaussian residuals have been developed previously. In such residuals, a reference model is confronted to data from the current system. In this paper, linear output-only systems depending on a varying external physical parameter are considered. These systems are driven by process noise, whose covariance may also vary between measurements. To deal with the varying parameter, an interpolation approach is pursued, where a limited number of reference models – each estimated from data measured in a reference state – are interpolated to approximate an adequate reference model for the current parameter. The problem becomes more complex when the different points of interpolation correspond to different noise conditions. Then conflicts may arise between the detection of changes in the eigenstructure due to a fault and the detection of changes due to different noise conditions. For this case, a new change detection approach is developed based on the interpolation of the eigenstructure at the reference points. The resulting approach is capable of change detection when both the external physical parameter and the process noise conditions are varying. This approach is validated on a numerical simulation of a mechanical system.

Keywords: Fault detection, changing process noise, subspace-based residual, model interpolation, linear parameter varying systems

1. INTRODUCTION

Vibration-based damage monitoring of mechanical structures aims at detecting changes in the dynamical behavior, and corresponds to detecting changes in the eigenstructure of a linear time-invariant (LTI) system [Carden and Fanning, 2004, Farrar and Worden, 2007]. When structures under operation are monitored, several practical issues, such as missing input signals and environmental variability, affect the choice of the monitoring procedure. The significance of environmental parameters, in particular of the temperature to the monitoring problem is described in [Sohn, 2007]. In order to handle such environmental effects, several approaches are proposed, e.g. regression analysis with respect to temperature effects [Peeters and De Roeck, 2001] or novelty detection with definition of the system under normalized reference conditions [Worden et al., 2002, Cross et al., 2012].

Data-driven stochastic subspace-based algorithms have shown to be appropriate for automated change detection, handling noise and uncertainties [Basseville et al., 2000, Viefhues et al., 2018]. Based on measurements under reference conditions of a structure, a reference null space is computed that is confronted to measurements of the test data in an asymptotic Gaussian residual and evaluated by hypothesis testing. Exposed to temperature changes, usually reference measurements at several reference temperatures are available. In this context, efforts have been undertaken to account for the variation of the temperature as an external parameter. Fritzen et al. [2003] assume constant reference models on intervals around each reference temperature. In [Balmès et al., 2008] a global reference model is built by averaging over all the reference measurements. Balmès et al. [2009] propose a statistical rejection approach where the system model is extended by a temperature effect model and corresponding sensitivities are considered to reject temperature changes as nuisance. In order to handle other environmental effects such as changes in the process noise covariance, Döhler and Mevel [2013], Döhler et al. [2014] introduced a robust formulation of the subspace-based residual. However, this approach still is sensitive to varying environmental parameters like the temperature.

Systems that depend on an external (environmental) parameter and that are linear for any fixed working point of this parameter, are well known in the concept of Linear Parameter Varying (LPV) models [Toth, 2010, Mohammadpour and Scherer, 2012]. Several system identification approaches exist for LPV models [van Wingerden and Verhaegen, 2009, Mercère et al., 2011, Lopes dos Santos et al., 2012]. This paper focuses on local model methods that are appropriate for the output-only state-space model structure. In local model approaches interpolation is a main issue. Besides methods that need coherence of the interpolated models [De Caigny et al., 2014, Zhang and Ljung, 2017], or address the interpolation of the output signals [Zhu and Xu, 2008], in [Zhang, 2018] a direct model interpolation of the local state-space models is proposed, which avoids the step of making the models coherent.

In this paper a change detection approach for linear systems subject to a combination of a varying external (physical) parameter and changing process noise covariance is proposed, inspired by the model interpolation approach in [Zhang, 2018]. A special application focus will be on mechanical structures, exposed to ambient excitation and temperature changes. On basis of [Zhang, 2018], in [Viefhues et al., 2019] a model interpolation procedure for the Hankel matrices of output covariances was derived. This approach and its sensitivity to changes in process noise covariance will be discussed first. Then a local model interpolation approach based on the local reference eigenstructures is developed for change detection at arbitrary working points, making use of the eigenstructure independence from the process noise covariance. The proposed procedure allows robustness to the effects of the varying external parameter as well as to changes in the process noise covariance.

The continuous-time mechanical model and its corresponding discrete time, linear time-invariant formulation are given in Section 2, where also the basics for hypothesis testing based on Gaussian residuals and subspace properties are recalled. The problem of changing conditions of the external parameter and process noise covariance, and their impact on the subspace residuals is expressed in Section 3, where a robust interpolation method is proposed. Finally, Section 4 presents the application of the method on a simulation of a mechanical system subject to changes in the ambient excitation and to external temperature variations.

2. FAULT DETECTION METHODOLOGY

2.1 State-space representation of mechanical systems

Let a linear time-invariant mechanical structure be affected by a scheduling parameter P that varies over time, such as the temperature. The vibration behavior at each working point P_j of that parameter can be described as a time-invariant system in continuous time t by

$$\mathbf{M}_j \ddot{z}_j(t) + \mathbf{C}_j \dot{z}_j(t) + \mathbf{K}_j z_j(t) = \nu_j(t), \tag{1}$$

where vector $z_j(t) \in \mathbb{R}^m$ contains the displacements at m degrees of freedom, and $\mathbf{M}_j$, $\mathbf{C}_j$ and $\mathbf{K}_j \in \mathbb{R}^{m\times m}$ are the mass, damping and stiffness matrices. The excitation of the system $\nu_j(t)$ is assumed to be unknown.

To motivate such modelling, assume there exists a relation between some physical parameter P and the set $(\mathbf{M}, \mathbf{C}, \mathbf{K})$. For example, it is known that the ambient temperature has a direct influence on the stiffness $\mathbf{K}$ [Balmès et al., 2009]. Thus, in general, any working point P_j corresponds to a set $(\mathbf{M}_j, \mathbf{C}_j, \mathbf{K}_j)$.

A discrete time-invariant state-space representation of the above mechanical structure valid at the j-th working point and sampled at time instants $t = k\tau$ can be written as [Juang, 1994]

$$\begin{cases} x_{j,k+1} = A_j x_{j,k} + w_{j,k} \\ y_{j,k} = C_j x_{j,k} + v_{j,k}, \end{cases} \tag{2}$$

where $x_{j,k} \in \mathbb{R}^n$ is the state vector and $y_{j,k} \in \mathbb{R}^r$ the corresponding output vector measured at r sensors, and $n = 2m$ is the model order. A_j and C_j denote the state transition and the observation matrices with

$$A_j = \exp\left(\begin{bmatrix} 0 & I \\ -\mathbf{M}_j^{-1}\mathbf{K}_j & -\mathbf{M}_j^{-1}\mathbf{C}_j \end{bmatrix} \tau\right) \in \mathbb{R}^{n\times n}$$

$$C_j = \begin{bmatrix} L_d - L_a \mathbf{M}_j^{-1}\mathbf{K}_j & L_v - L_a \mathbf{M}_j^{-1}\mathbf{C}_j \end{bmatrix} \in \mathbb{R}^{r\times n},$$

where $L_d, L_v, L_a \in \{0,1\}^{r\times m}$ are selection matrices depending on the positions and types of sensors. The process noise $w_{j,k}$ of the output-only system is unknown and assumed to be white noise with constant covariance for each dataset, but possibly different covariance between different datasets, and $v_{j,k}$ denotes the measurement noise.

Let θ be a system parameter vector, which describes the properties of the dynamical system (1) or (2) completely. For mechanical systems, an adequate parameterization may be the eigenstructure, or the collection of physical parameters like element masses and stiffnesses. In the (unfaulty) reference state the system parameter is assumed to be $\theta_{j,0}$ for each working point P_j, and changes in the system will be reflected in changes of the system parameter vector $\theta \neq \theta_{j,0}$.

2.2 Data-driven residual function for fault detection

A stochastic subspace-based fault detection method for detecting changes in θ has been proposed in [Basseville et al., 2000]. From N data samples of the output signal in the reference state $\mathcal{Y}_{j,N} = \{y_{j,1}, y_{j,3}, ..., y_{j,N}\}$, the output covariances $R_{j,i} = \mathbf{E}(y_{j,k+i}\, y_{j,k}^T) = C_j A_j^{i-1} G_j$, $G_j = \mathbf{E}(x_{j,k+1}\, y_{j,k}^T)$ are computed. Consequently, the block Hankel matrix $\mathcal{H}_j \in \mathbb{R}^{(p+1)r\times qr}$ of the output covariances

$$\mathcal{H}_j \stackrel{\text{def}}{=} \begin{bmatrix} R_{j,1} & R_{j,2} & \dots & R_{j,q} \\ R_{j,2} & R_{j,3} & \dots & R_{j,q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{j,p+1} & R_{j,p+2} & \dots & R_{j,p+q} \end{bmatrix} \stackrel{\text{def}}{=} \text{Hank}(R_{j,i}), \tag{3}$$

is defined, which is related to the system matrices A_j and C_j and thus to the system parameter θ thanks to its factorization property

$$\mathcal{H}_j = \mathcal{O}_j \mathcal{C}_j \tag{4}$$

into observability and controllability matrices

$$\mathcal{O}_j = \begin{bmatrix} C_j \\ C_j A_j \\ \vdots \\ C_j A_j^p \end{bmatrix} \text{ and } \mathcal{C}_j = \begin{bmatrix} G_j & A_j G_j & \dots & A_j^{q-1} G_j \end{bmatrix}. \tag{5}$$

Considering the left null space $S(\theta_{j,0})$ of the block Hankel matrix, it holds

$$S(\theta_{j,0})^T \mathcal{H}_j = 0, \quad S(\theta_{j,0})^T \mathcal{O}_j = 0. \tag{6}$$

Based on this property, a residual function ζ is computed from the left null space and a current data set $\mathcal{Y}_{j,N}$ as

$$\zeta(\theta_{j,0}, \mathcal{Y}_{j,N}) \stackrel{\text{def}}{=} \sqrt{N}\, \text{vec}(S(\theta_{j,0})^T \widehat{\mathcal{H}}_j), \tag{7}$$

where $\widehat{\mathcal{H}}_j = \text{Hank}(\widehat{R}_{j,i})$, $\widehat{R}_{j,i} = \frac{1}{N}\sum_{k=1}^{N} y_{j,k+i}\, y_{j,k}^T$ obtained as a consistent estimate from the testing data. The expectation of (7) when using testing data from systems operating at parameter θ is

$$\mathbf{E}_\theta(\zeta(\theta_{j,0}, \mathcal{Y}_{j,N})) = 0 \quad \text{iff} \quad \theta = \theta_{j,0}. \tag{8}$$

The asymptotic approach for change detection [Benveniste et al., 1987] is used for a statistical evaluation of the residual in (7). With the close hypotheses

$$\begin{array}{lll} \mathbf{H}_0^j: & \theta = \theta_{j,0} & \text{(reference system)}, \\ \mathbf{H}_1^j: & \theta = \theta_{j,0} + \delta_j/\sqrt{N} & \text{(faulty system)}, \end{array} \tag{9}$$

the central limit theorem applies for the residual [Basseville et al., 2000] and for $N \to \infty$ it holds

$$\zeta(\theta_{j,0}, \mathcal{Y}_{j,N}) \longrightarrow \begin{cases} \mathcal{N}(0, \Sigma_j) & \text{under } \mathbf{H}_0^j \\ \mathcal{N}(\mathcal{J}_j\, \delta_j, \Sigma_j) & \text{under } \mathbf{H}_1^j \end{cases} \tag{10}$$

where the vector δ_j is unknown but fixed and $\mathcal{J}_j$ denotes the sensitivity of the residual with respect to the system parameter θ evaluated at $\theta_{j,0}$ [Döhler and Mevel, 2013]. Σ_j is the residual covariance. A faulty system is hence characterized by a deviation of the residual mean from zero.

To test if the residual in (10) corresponds to the reference system with $\theta = \theta_{j,0}$ or to the faulty system with $\theta \neq \theta_{j,0}$, the corresponding generalized likelihood ratio (GLR) test writes [Basseville, 1997]

$$t_j = \zeta_j^T \Sigma_j^{-1} \mathcal{J}_j (\mathcal{J}_j^T \Sigma_j^{-1} \mathcal{J}_j^T)^{-1} \mathcal{J}_j^T \Sigma_j^{-1} \zeta_j, \tag{11}$$

where $\zeta_j = \zeta(\theta_{j,0}, \mathcal{Y}_{j,N})$. The test statistic is asymptotically χ^2-distributed with non-centrality parameter $\delta_j^T F_j \delta_j$ in the faulty state, where $F_j = \mathcal{J}_j^T \Sigma_j^{-1} \mathcal{J}_j^T$ is the Fisher information. Hence, a threshold can be set to decide between reference and faulty states.

3. FAULT DETECTION OF SYSTEMS UNDER CHANGING CONDITIONS

3.1 Motivation

When it comes to fault detection in LPV systems, the variation of the scheduling parameter itself leads to changes in the system's dynamical characteristic. Consider a system with a scheduling parameter P that is different from the known working points P_j, $j = 1, \ldots, u$, for which data from a reference state is available. In this case, possibly none of these working points describes an appropriate reference state for the new working point P, and the above described fault detection methodology will be inadequate for any j. The basic idea of the proposed fault detection approach is to obtain a well fitting reference null space

$$\underline{S}(\theta_0(P)) \tag{12}$$

for the given parameter P by using interpolation.

3.2 Local model interpolation of LPV systems with constant process noise

In [Zhang, 2018] a local model interpolation approach for LPV systems is proposed, where a large global state-space model is built from local models, defined from data measured at u given working points of the scheduling parameter. For any parameter value P, the global model is defined as

$$\begin{aligned} \underline{x}_{k+1} &= \underline{A}\, \underline{x}_k + \underline{w}_k \\ y_k &\overset{\text{def}}{=} \underline{C}(P)\, \underline{x}_k + \underline{v}_k\,, \end{aligned} \tag{13}$$

where

$$\underline{A} \overset{\text{def}}{=} \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_u \end{bmatrix}, \ \underline{x}_k \overset{\text{def}}{=} \begin{bmatrix} x_{1,k} \\ \vdots \\ x_{u,k} \end{bmatrix}, \ \underline{w}_k \overset{\text{def}}{=} \begin{bmatrix} w_{1,k} \\ \vdots \\ w_{u,k} \end{bmatrix},$$

$$\underline{C}(P) \overset{\text{def}}{=} [\rho_1(P) C_1 \ \cdots \ \rho_u(P) C_u]\,,$$

$$\underline{v}_k \overset{\text{def}}{=} \sum_{j=1}^{u} \rho_j(P)\, v_{j,k}(t)\,.$$

The weighting functions $\rho_j(P)$ can be chosen e.g. as bell-shaped functions, when a section-wise approximated linear effect of the scheduling parameter can be assumed. The function is then centered at P, yielding

$$\sum_{j=1}^{u} \rho_j(P) = 1 \tag{14}$$

for any P, and $0 \leq \rho_j(P) \leq 1$. On the basis of this interpolation approach, an appropriate null space $\underline{S}(\theta_0(P))$ can be obtained as follows. This null space should be the left null space of the Hankel matrix $\underline{\mathcal{H}}(P)$ of output covariances at the working point P in the reference state. The output covariances $\underline{R}_i(P) = \mathbf{E}(y_{k+i} y_k^T)$ of system (13) yield [Viefhues et al., 2019]

$$\begin{aligned} \underline{R}_i(P) &= \underline{C}(P) \underline{A}^{i-1} \underline{G}(P) \\ &= \sum_{j=1}^{u} \rho_j^2(P) C_j A_j^{i-1} G_j = \sum_{j=1}^{u} \rho_j^2(P) R_{j,i}\,, \end{aligned} \tag{15}$$

with $\underline{G}(P) = \left[\rho_1(P) G_1^T \ \ldots \ \rho_u(P) G_u^T\right]^T$. This leads to the interpolated Hankel matrix

$$\underline{\mathcal{H}}(P) = \sum_{j=1}^{u} \rho_j^2(P) \mathcal{H}_j. \tag{16}$$

From $\underline{\mathcal{H}}(P)$, the interpolated reference null space $\underline{S}(\theta_0(P))$ can be recovered with a singular value decomposition. The residual $\zeta(\theta_0(P), \mathcal{Y}_N(P))$, which uses the interpolated reference null space and a current data set at parameter P, then writes as

$$\zeta(\theta_0(P), \mathcal{Y}_N(P)) \overset{\text{def}}{=} \sqrt{N}\, \text{vec}(\underline{S}(\theta_0(P))^T \widehat{\mathcal{H}}(P)) \tag{17}$$

In the statistic test, the covariance $\Sigma(P)$ of the residual $\zeta(\theta_0(P), \mathcal{Y}_N(P))$ is computed anew for a given P from the data covariances of the test dataset $\mathcal{Y}_N(P)$. The central limit theorem should apply for the residual,

$$\zeta(\theta_0(P), \mathcal{Y}_N(P)) \longrightarrow \begin{cases} \mathcal{N}(0, \Sigma(P)) & \text{under } \mathbf{H}_0^P \\ \mathcal{N}(\delta_P, \Sigma(P)) & \text{under } \mathbf{H}_1^P\,, \end{cases} \tag{18}$$

where $\mathbf{H}_0^P$ and $\mathbf{H}_1^P$ are properly defined with respect to damage δ_P and P. For this residual, no sensitivity is computed, which is not preventing to detect a change in the mean of the designed residual, and the GLR melts down to

$$t_P = \zeta^T (\Sigma(P))^{-1} \zeta. \tag{19}$$

This local model interpolation is valid only if constant process noise covariance can be assumed at the different working points P_j in the reference state, otherwise the output interpolation in (13) would not be coherent. Then, datasets from reference working points under low process noise covariance would be considered less in the interpolation compared to datasets under high process noise covariance, since the weightings $\rho_j(P)$ only take into account the parameters P_j and P. Thus, the interpolation of the output covariances in (16) will not be successful for the definition of a meaningful left null space $\underline{S}$ in this case.

3.3 Model interpolation robust to changes in the process noise covariance

Inspired by the local model interpolation method in [Zhang, 2018], a fault detection method for linear parameter varying systems with changes in the process noise covariance at different working points is developed.

Instead of computing $\underline{S}(\theta_0(P))$ on the interpolated Hankel matrix $\underline{\mathcal{H}}(P)$ to satisfy the condition $\underline{S}(\theta_0(P))^T\underline{\mathcal{H}}(P) \approx 0$ in the reference state, we propose to compute an interpolated parametric observability matrix $\underline{\mathcal{O}}(\theta_0(P))$, having the analogous property $\underline{S}(\theta_0(P))^T\underline{\mathcal{O}}(\theta_0(P)) \approx 0$ in the reference state, see (6). To this end, the eigenstructure of the system is identified at each reference working point P_j, since the observability matrix $\mathcal{O}(\theta_{j,0})$ is fully defined from it. Moreover, the eigenstructure is a canonical parameterization that can be easily interpolated between different working points, and its estimates are independent from the process noise covariance.

For each working point P_j, the eigenstructure is defined as the collection of eigenvalues $\lambda_{j,l}$ of A_j and observed eigenvectors $\varphi_{j,l}$ with

$$A_j\,\phi_{j,l} = \lambda_{j,l}\,\phi_{j,l}\,,\; l = 1, 2, ..., n \tag{20}$$

$$\varphi_{j,l} = C_j\,\phi_{j,l}\,,\; l = 1, 2, ..., n. \tag{21}$$

They can be estimated from the output data e.g. with subspace-based system identification [Van Overschee and De Moor, 1996]. For simplicity of notation, assume that the system parameter θ itself consists of the eigenstructure, i.e. at the working points in the reference state it yields

$$\theta_{j,0} \overset{\text{def}}{=} \begin{bmatrix} \Lambda_j \\ \text{vec}(\Phi_j) \end{bmatrix}$$

where $\Lambda_j = [\lambda_{j,1}\ \lambda_{j,2}\ \ldots\ \lambda_{j,n}]^T$, $\Phi_j = [\varphi_{j,1}\ \varphi_{j,2}\ \ldots\ \varphi_{j,n}]$.

To obtain the interpolated reference parameter $\theta_0(P)$ at a new working point P from $\theta_{j,0}$, $j = 1, \ldots, u$, it has first to be made sure that the estimated observed eigenvectors have a comparable scaling, since they can only be identified up to a constant. Assuming small eigenstructure changes between the different working points, a coherent scaling can be obtained with respect to one chosen reference working point j^* by finding the scaling factors $\alpha_{j,l} \in \mathbb{C}$ such that

$$\alpha_{j,l}\varphi_{j,l} \approx \varphi_{j^*,l}\,,\text{ for } j = 1, ..., u\,,\; l = 1, \ldots, n,$$

thus

$$\alpha_{j,l} = \frac{\varphi_{j,l}^H \varphi_{j^*,l}}{\varphi_{j,l}^H \varphi_{j,l}}. \tag{22}$$

Finally, the eigenstructure can be interpolated for a given scheduling parameter value P as

$$\underline{\theta}_0(P) = \sum_{j=1}^{u} \rho_j(P)\,\theta_{j,0}. \tag{23}$$

From $\underline{\theta}_0(P)$, the parametric observability matrix $\underline{\mathcal{O}}(\theta_0(P))$ can be computed [Döhler et al., 2014] and its left null space $\underline{S}(\theta_0(P))$ can be recovered with a singular value decomposition such that

$$\underline{S}(\theta_0(P))^T\underline{\mathcal{O}}(\theta_0(P)) = 0. \tag{24}$$

The underlying interpolation is independent from the process noise covariance in the reference state, since the eigenstructure estimates at the working points P_j are not affected, while the matrices $\mathcal{H}_j$ in (16) are. This leads to the residual for linear parameter varying systems $\zeta(\theta_0(P), \mathcal{Y}_N(P))$ that is robust to process noise changes, and the corresponding hypothesis test can be derived similarly as in (19), based on the asymptotic properties in (18). Notice that here the covariance $\Sigma(P)$ has to be computed on the test data, i.e. on $\mathcal{Y}_N(P)$, due to possible changes in the noise properties.

4. NUMERICAL APPLICATION

The developed robust fault detection method is applied to the eight mass-spring-damper system in Figure 1, where the masses m and stiffnesses k are defined as $m_i = 1$, $k_i = 200$ for $i = 1, 3, 5, 7$, and $m_i = 2$ and $k_i = 100$ for $i = 2, 4, 6, 8$. All modes have a damping ratio of 2%. A fault of the system is simulated as a decrease of stiffness by 2% at all elements. The varying parameter of the system is the temperature. An increase in temperature results in a nonlinear decrease of the stiffness, similar to considerations in [Bhuyan et al., 2019].

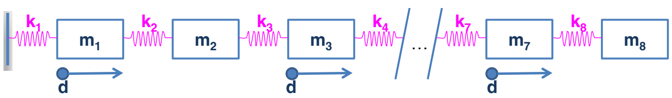

Fig. 1. Mass-spring-damper system.

The system excitation is modeled by a Gaussian white noise input signal at input positions located at masses m_1, m_3, m_5, and m_7. Changes in the process noise are considered by varying randomly the covariance of the excitation. The output signal is recorded at four displacement sensors, located at the input positions. The signal is sampled with 10 Hz, and white measurement noise with 5% standard deviation of the output is added. 160000 sampling points are considered to built the block Hankel matrices at each working point, that is at each reference temperature, for the computation of the interpolated reference null space $\underline{S}$. The output signals for the tests are simulated with 200000 data points.

In the following two examples, the test statistic is calculated for testing temperatures between 0°C and 20°C, both for the system in reference and faulty states.

Firstly, the process noise covariance is assumed to be constant. The influence of the varying temperature parameter is illustrated in Figure 2. The left plot shows the test values t from (19), when all the reference output data is recorded at a reference temperature 10°C. Thus, the computed reference null space only fits to monitoring data from this reference temperature. For unchanged systems at other temperatures, this leads to higher test values due to system change, leading to false alarms. On the other hand, the test statistic of the faulty system at temperatures

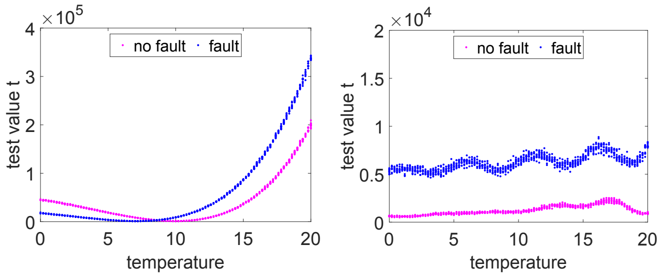

Fig. 2. Test statistic for varying parameters and constant process noise covariance: conventional computation (left) and local model interpolation algorithm from Section 3.2 (right).

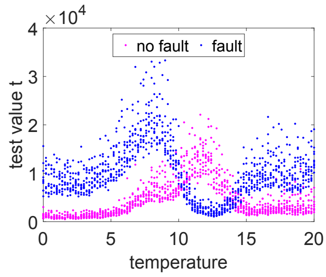

Fig. 3. Test statistic for local model interpolation approach from Section 3.2 with changes in the process noise covariance.

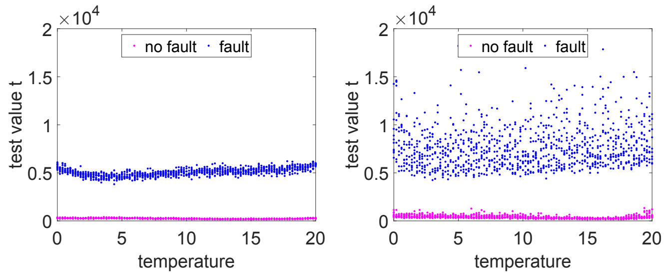

Fig. 4. Test statistic for varying parameters using the interpolated eigenstructure with the algorithm of Section 3.3, for constant process noise covariance (left) and changing process noise covariance (right).

below the reference temperature of 10°C is lower than the values of the reference state, leading to false negatives, while it is above the values of the reference state at higher temperatures. This phenomenon can be explained from the similar effects of temperature and damage to the mechanical system, since both change the stiffness. Consequently, it is not possible to define a reasonable threshold between the values of the reference and faulty state due to the varying temperature parameter.

For this LPV system, the application of local model interpolation from Section 3.2 allows for computation of an adequate reference for each testing temperature. In the reference state output data is recorded at five reference temperatures $0, 5, 10, 15$, and 20° C. The weighting function is chosen to be a centered Gaussian function with standard deviation of $10/3$. The interpolated reference left null space from (16) is computed for the temperature in the test state and confronted to the test data by using (17). The data covariance matrix Σ is built directly in the test state from the test data.

The test values of the local model interpolation approach (Figure 2, right) increase slightly with the temperature, having some variation between the reference working points. It can be concluded that the effect of the temperature parameter on the output covariances is not linear. Still, the interpolated reference seems to fit quite well for all testing temperatures, as the test values for the reference and the faulty system are clearly separated.

The previous results apply when the process noise covariance is constant for the working points of the reference state. In real applications in the structural engineering field this may not hold. Figure 3 illustrates the effect, when the standard deviation of the excitation in the reference state at 10°C is changed from 1 to 0.2, and the standard deviation of the excitation in the test state is uniformly randomly varied between 1 and 10 for each input. In this setting no successful fault detection is possible, since the test statistics of the reference and faulty states cannot be separated. The interpolation of output data from a system with changing excitation properties is indeed not valid and leads to inadequate reference null spaces.

This problem can be overcome by means of the developed method robust to changing process noise covariance in Section 3.3, which uses the interpolated eigenstructure of the system. With data recorded at the five reference temperatures, the five sets of temperature depending system parameters are recovered with stochastic subspace-based system identification [Van Overschee and De Moor, 1996]. The identified observed eigenvectors (mode shapes) are scaled to the mode shape at temperature 10°C, and interpolated for the temperature in the test state with (23). The weighting function is chosen as above. The observability matrix $\underline{\mathcal{O}}(\theta_0(P))$ is computed on the interpolated eigenstructure to finally obtain the reference null space for the testing temperature with a SVD, see (24).

Figure 4 (left) shows the test values of the LPV system for the interpolated eigenstructure when no changes in the process noise covariance occur. The unchanged and the changed system state can be clearly identified. Compared to the local model interpolating approach in Figure 2 (right), the test values are more stable, at the price of the additional system identification at the reference working points. It seems indeed that the varying temperature parameter affects the eigenstructure approximately linearly locally for the considered temperatures in the test states.

When considering changes in the process noise covariance (Figure 4, right), those changes are reflected in the variance of the test values in the unchanged and in the faulty system. The standard deviation is of about 30% of the mean test value. The different system states again are well separated and fault detection is successful.

5. CONCLUSION

In this paper, a fault detection method for linear systems has been proposed to handle nuisances regarding changes of the process noise covariance as well as changes of an external physical parameter. The main difference between both nuisances is the availability of measurements of the physical parameter, which allows to build a collection of reference states at different physical working points. The proposed method interpolates the eigenstructure of the linear system at any working point based on the values of the physical reference points, and allows also to be robust to changes in the process noise statistics. The robustness towards process noise changes has become possible by system identification in the reference state for the set of physical reference points in order to retrieve the eigenstructure, however no further system identification is necessary on the test data.

Besides application to real data, future work includes the consideration of the uncertainty of the estimates in the reference state, similarly as in [Viefhues et al., 2018]. Moreover, the sensitivity $\mathcal{J}(P)$ will be evaluated by interpolating the eigenstructure for any given value of P in order to perform fault isolation.

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